Oral-Example-Geometry-Topology

1 Personal

1. Let $\pi : S^3 \to S^2$ be the Hopf fibration. Recall that if we identify $S^3 = \{(z_1, z_2) \in \mathbb{C}^2 || z_1|^2 + |z_2|^2 = 1\}$ and $S^2 = \mathbb{CP}^1$ with homogenous coordinate $[z_1, z_2]$, then

$$\pi : (z_1, z_2) \to [z_1, z_2].$$

   Show that there doesn't exist a section for π, i.e., a smooth map $s : S^2 \to S^3$ such that $\pi \circ s : S^3 \to S^2$ is the identity.

2. Show that there exists no degree one map from $S^2 \times S^2$ to $\mathbb{CP}^2$.

3. Show that there exists no metric on $\mathbb{RP}^2 \times \mathbb{RP}^2$ with positive sectional curvature.